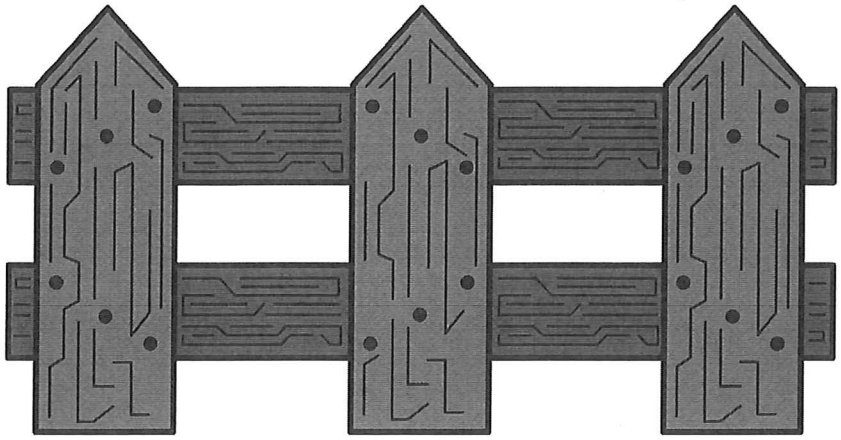
Painting the fence



Ben and Cleo can paint a fence in four days. Anna and Ben can do it in two days. Anna and Cleo can paint it in three days. How many days, as a fraction, does it take all of them working together, if Cleo gets injured at the end of the first day and can't come back to work?

From Alberta High School Mathematics competition 2017

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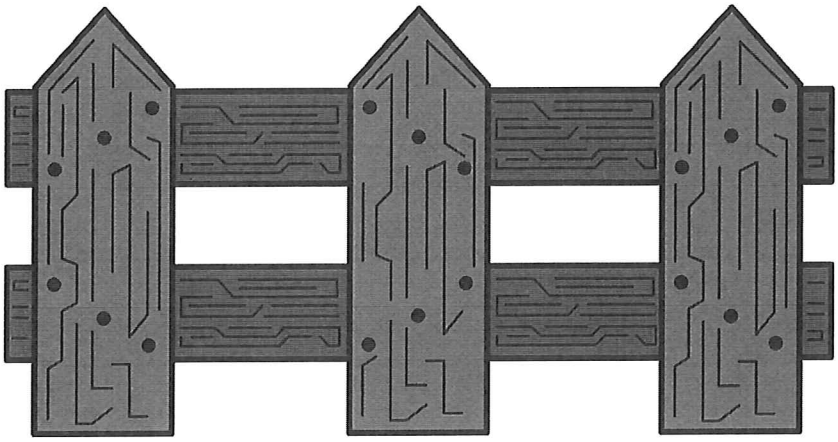

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SOLUTION

$B+C = 4$ days or $\frac{1}{4}$ in 1 day $= \frac{6}{24}$

$A+B = 2$ days or $\frac{1}{2}$ in 1 day $= \frac{12}{24}$

$A+C = 3$ days or $\frac{1}{3}$ in 1 day. $= \frac{8}{24}$

$A = \frac{8}{24} - C$

First day all 3 painted

$A + B + C = \frac{6}{24} + (\frac{8}{24} - C)$

($\frac{8}{24} - C$) ($\frac{6}{24}$)

$A + B + 2C = \frac{14}{24}$

($\frac{12}{24}$)

$2C = \frac{14}{24} - \frac{12}{24}$

$2C = \frac{2}{24}$

$C = \frac{1}{24}$

So $A+B+C = \frac{13}{24}$ on day 1.

Still need to do $\frac{11}{24}$ of the job with only A+B painting. A+B can do $\frac{12}{24}$ in 1 day

$\frac{11}{24} \div \frac{12}{24} = \frac{11}{12}$ of a day.

Therefore, it will take $\frac{11}{12} + \frac{12}{12} = \frac{23}{12}$ days to paint the fence.

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