Curricular content

Operations with integers Addition, subtraction, multiplication, division AND order of operations

Order of operations includes brackets but does not include exponents

Extends to decimals as well

Examples and Strategies

Always start with manipulatives. Introduce with algebra tiles or if you don't have them, two sided counters or even colored tiles.





rearrange into zero pairs



$$(-4) + (-3) =$$





you cannot make zero pairs so the answer is -7

Subtraction with integers







Start with -5 (five red tiles). The question asks you to subtract 3 positive tiles but you don't have any positive tiles available. Therefore you need to create zero pairs and add them to your set. This is why it is important to understand that adding zero to a number does not change the quantity.

Once you have added zero pairs, you can then subtract three positive tiles, leaving you with a total of 8 red tiles, or -8

Language

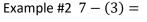
Zero pair: a Pair of numbers that when added together equals zero. Example -1 and 1. This concept is essential for future equations work.

Negative:

 Please do not teach that two negatives makes a positive. This is not always true and can interfere with understanding.

Order of operations:

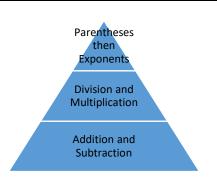
BEDMAS: or PEDMAS Brackets (or parentheses), Exponents, Division and Multiplication in the order they occur in the equation, Addition and Subtraction in the order they occur in the equation. Grade 7 does NOT include exponents.







Start with 7 positive tiles. You are asked to subtract three negative tiles. In order to do so, you must create zero pairs. Then you can subtract the negative tiles, leaving you with 10 positive tiles, or +10.



The above diagram can make it easier to understand the order in which we do the operations. It is hierarchical. In general terms we can think of it as the operation that has the greatest potential impact on the final answer is done, and ends with the operation with generally the least impact (adding and subtracting)

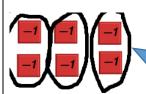


Start by thinking of multiplication as "groups of". When you multiply, start with ZERO on the table and then add your groups to the table as shown in the diagram.



 $3 \times 2 = 6$

3 groups of positive 2 is 6

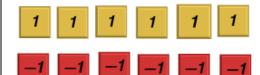


 $3 \times (-2) = -6$

Three groups of negative 2 is negative 6



 $(-3) \times 2 = -6$ Even though we know that this is the same as $2 \times (-3) = -6$, it is important to illustrate it this way as well. When multiplying remember we started with "zero" on the table and then added our groups to the table. In this case we are looking now to "remove" groups. In order to have groups to remove, we need to create zero pairs. Once you have placed the zero pairs you can then remove 3 groups of positive 2, leaving you with -6.



 $(-3) \times (-2) = 6$ This is very similar to the question above. In order to "remove" three groups of -2, you need to create zero pairs to start. Then remove three groups of -2 and you will be left with positive 6.

Where does this lead?

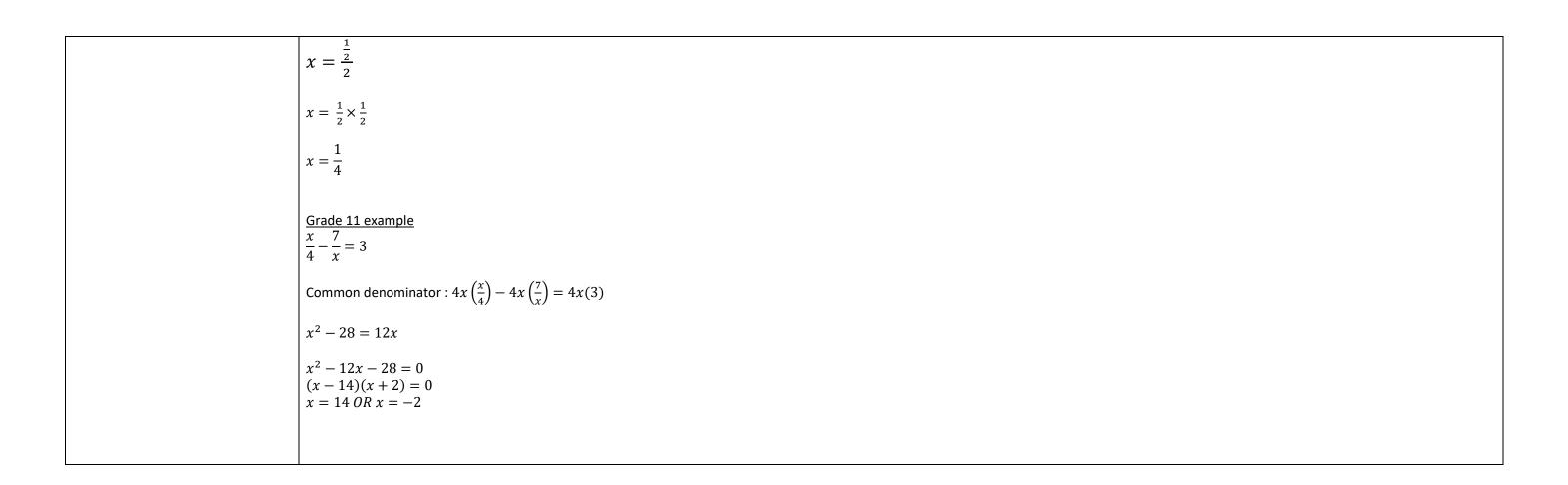
Essential for algebra Grade 9 example:

$$2x + \frac{1}{10} = \frac{3}{5}$$

Subtract $\frac{1}{10}$ from each side (zero pairs)

$$2x = \frac{6}{10} - \frac{1}{10}$$

$$2x = \frac{5}{10}$$



Critical concept: **Equations**

Curricular content

2 step equations with whole number coefficients, constants and solutions

Preservation of equality

Examples and Strategies

Difference between an expression and an equation

Expression	Equation
Can be simplified	Can be solved
Doesn't contain =	Contains =
Cannot determine	Can determine the value of
numerical value of variable	the variable

Example #1

Create an expression to represent 5 more than a number x+5

Language

Equal means "balance" or the same

Constant: symbol with a fixed numerical value. (does not change)

Variable: symbol which represents an unknown value e.g. in the equation $3x + 4 = 19 \, x$ represents the variable, 4 is the constant and 3 is the coefficient

Coefficient: number that multiplies the variable e.g. in 4x, 4 is the coefficient and the variable x is multiplied by 4.

Preservation of equality: understand the equality is a relationship rather than an "operation". This means that what is on each side of the equal sign must be equal. Therefore, we use compensation to keep the quantities equal. For example, if we add a quantity to one side of the equation, we must add the same to the other side. If we multiply,

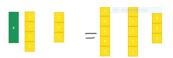
ii) 3 more than twice a number

2n + 3

Equation examples

$$\begin{array}{c}
 x + 7 = 13 \\
 x + 7 - 7 = 13 - 7 \\
 x = 6
 \end{array}$$

Shown with algebra tiles

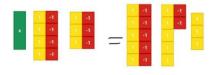


Step one: the green bar is the variable. The yellow squares are the constant (+1). Create your equation.

Important tips for algebra: we use letters to represent the unknown numbers.

Don't always use the same letter to represent the variable

Letters are written in cursive in order to differentiate between operations and variables

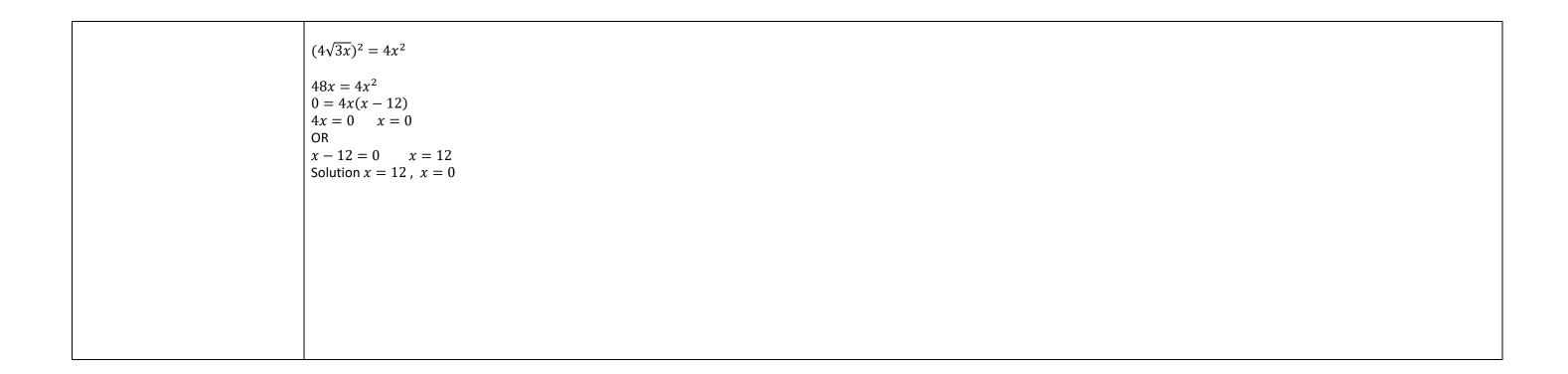


Step 2: Isolate the variable by creating zero pairs for the constants. Since you need seven (-1) tiles to isolate the variable on the left side of the equation, you also add 7 (-1) tiles to the right side of the equation in order to keep the balance.



Step 3: Since you only have one green bar (a single variable) you can see that $x=6\,$

3x - 4 = 8subtract or divide, the same 3x - 4 + 4 = 8 + 4principle applies. 3x = 12Zero pairs: example (-2) and 2 Used to help isolate variable in an $\frac{3x}{3} = \frac{12}{3}$ equation x = 4With algebra tiles Step 1: create the equation. Step 2: Isolate the variable by adding zero pairs (in this case 4 (+1) tiles) to the left side of the Step2 equation. In order to keep the balance, add the same number of (+1) tiles to the right side of the equation. Step 3: You can see that 3x = 12. We want to know the value of a single x so we divide the left and the right side of the equation into three equal groups. From here we can see that x = 4Step3 Where does this lead? Grade 11 example: Solving equations with radicals $7 + \sqrt{3x} = \sqrt{5x + 4} + 5$ $2 + \sqrt{3x} = \sqrt{5x + 4}$ $=(2+\sqrt{3x})^2 = (\sqrt{5x+4})^2$ $=4 + 4\sqrt{3x} + 3x = 5x + 4$ Isolate the radical $=4\sqrt{3x}=2x$ Square both sides



Curricular content

Relationship between decimals, fractions, ratio and percent -conversions -equivalency

-terminating/repeating decimals -comparing and ordering decimals and fractions using a number line

-financial: % calculations, discounts, tips, sale price

Examples and Strategies

Conversions between fractions, ratios, percent, decimals: students should be able to convert from one representation to others Example #1

45%,
$$\frac{45}{100}$$
, 0.45

Know the following repeating decimals 1

$$\frac{1}{3} = 0.\,\overline{3}$$

$$\frac{2}{3} = 0.\overline{6}$$

Repeating decimals teaching tip

Terminating decimal demonstration: take a hundreds grid and divide it evenly by 4 (shared amongst 4 people). Each person gets 25 squares.

Repeating decimal: take the same hundreds grid and share it evenly with 3 people. Everyone will get 33 squares with one square left. Cut that square into 10 pieces (because we are dealing with decimals so each place you move to the right will be ten times smaller) and share it evenly with those three people. You will still have one square left that you need to cut into ten more tiny pieces. This will continue forever. This is a repeating decimal.

Language

Terminating decimals: decimal with a finite number of digits. Can be rewritten as a fraction. Is a rational number.

Repeating decimal: decimal repeats e.g. 0.123123123... or 0.444.... Is a rational number

Writing a repeating decimal $0.\overline{123}$ Or if it is a single digit that repeats $0.\dot{4}$

Proper language when naming numbers with decimals: 0.25 "25 hundredths" rather than "point two five"
Example #2: 14.2 is "14 and 2 tenths"

-make sure to always write the zero in front of the decimal (e.g. 0.25 instead of .25)

Finding missing parts:

Find 3% of 1800

Two methods: Method #1 Find 10% of 1800 which is 180 Find 1% which is 18 3% is $3\times18=54$

Example #2

Find 15% of 80 $0.15 \times 80 = 12$

Example #3

18 is 6% of what number? 18 = 0.06x $\frac{18}{0.06} = x$ x = 300

Method #2 Find 3% of 1800

Relate to multiplication for example 3 groups of 15 is 45

3% means 0.03

3% of 1800 means 0.03 groups of 1800 or $0.03 \times 1800 = 54$

Percentage discounts

Finding percentage discounts is similar to the examples shown above. Once the discount is calculated, it must be subtracted from the original price in order to determine the new price. This is a common oversight.

Example: Your favourite clothing store is having a sale on jeans. The regular price of the jeans is \$84 and they are now on sale for 20% off. What is the new price, before taxes?

Steps:

- 1) Calculate 10% of \$84 which is \$8.40.
- 2) 20% will then be $2 \times \$8.40 = \16.80
- 3) Since \$16.80 is the discount, we then need to subtract this from the original purchase price of \$84

\$84 - \$16.80 = \$67.20

Therefore, the final price, before taxes, is \$67.20

Or method #2

Find 20% of \$84

 $0.2 \times \$84 = \16.80

The discount is \$16.80 which must then be subtracted from the original price.

\$84 - \$16.80 = \$67.20

Where does this lead?

Workplace Math 11:

Sally is charged \$0.35 every time she uses her debit card for purchases under \$5.00

On Monday she used her debit card three times. She spent \$4.75 at Starbucks in the morning and \$3.50 at Tim Hortons for lunch and \$2.25 for an afternoon coffee.

- a) How much did Sally spend on food?
- b) What would be the bank charge for her debit card usage?
- c) What percent of her total amount spent was the debit charge surcharge?

SOLUTION

- a) \$10.50 was spent on food
- b) \$1.05 was spent on debit card surcharge
- c) 9% of the total was due to debit card surcharges