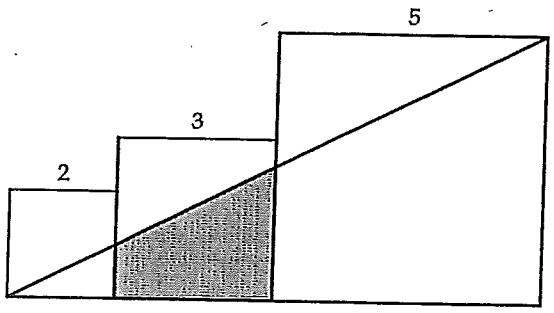
Quadrilateral

Three squares have dimensions as indicated in the diagram. What is the area of the shaded quadrilateral?



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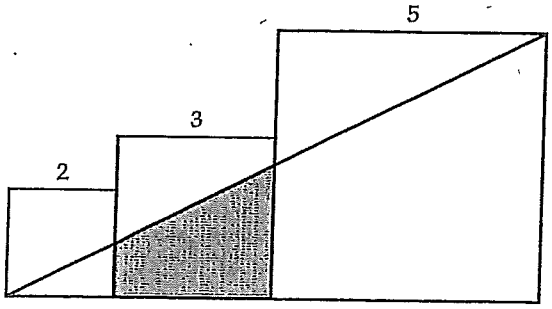

9. Let x and y denote the lengths of the sides of the shaded region. Since ratios of corresponding sides of similar triangles are equal, we have

$$\frac{5}{10} = \frac{x}{2} = \frac{y}{5}$$

from which it follows $x = 1$ and $y = \frac{5}{2}$. The area of the shaded region is the difference between the areas of two triangles: one with height $y = \frac{5}{2}$ and base 5 and the other with height $x = 1$ and base 2. This gives

$$\text{area of shaded region} = \frac{1}{2}(5)\left(\frac{5}{2}\right) - \frac{1}{2}(1)(2) = \frac{25}{4} - 1 = \frac{21}{4}$$

Alternative solution:
The area of the shaded region is the area of a trapezoid with the two parallel sides with lengths $x = 1$ and $y = \frac{5}{2}$ and height 3. Hence,

$$\text{area of shaded region} = \frac{3}{2}\left(1 + \frac{5}{2}\right) = \frac{3}{2} \times \frac{7}{2} = \frac{21}{4}$$

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