Grade : 5

Critical concept: **Multiplication double (or triple) digit and area and perimeter**

Curricular content

Multiplication 3 digits (for this page we will do 2 digit by 2 digit but you will see how to extend to three digits)

Area: finding the area of squares and rectangles (which is exactly multiplication☺)

Relating area and perimeter

Language

Factor: side length of the rectangle in the area model

Factors are multiplied to form the product

Partial product: when you have decomposed the shape or number into smaller parts, you determine the area of that part. When you add all the partial products together you will have the final product.

Area: the amount of square units it would take to cover the space/shape. Area is two dimensional (length x width)

Perimeter: distance around the outside of the shape. Perimeter is one dimensional (linear-adding length of sides around the outside)

Examples and Strategies

45 x 23=

	40	5
20	800	100
3	120	15

Add the partial products

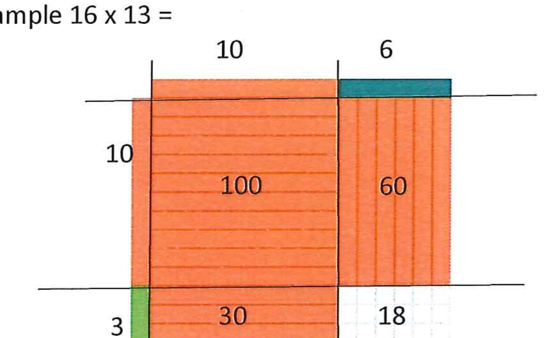
800 + 100 + 120 + 15 = 1035

<u>For specific tips on teaching multiplication please see the grade 4 critical concept sheet.</u>

It is really important that students understand the area model of multiplication and that they are able to explain the concept product, and name it as showing the area. Drawing the diagram as an approximation as shown above helps students see how many unit tiles (think of using the multi-coloured tile manipulatives) would be covering the rectangle made by 45 columns, 23 rows.

It can be very helpful to use Cuisenaire rods to show this same concept, as counting out the individual tiles can be very time consuming with these larger numbers.

Example 16 x 13 =



Keep the factors on the outside- which can be removed once you've filled the rectangle to the appropriate size- you will see the partial products more visually

16 x 13=

	10	6
10	100	60
3	30	18

Partial Products

100+60+30+18= 208

Note to teachers: The orange Cuisenaire rods have a value of 10, dark green value = 6, light green Cuisenaire rods have a value of 3. The white/light grey Cuisenaire rods have a value of 1. This makes it easy to see the partial products at a glance.

Once students really understand the area model of multiplication, it is much easier to teach finding the area of squares and rectangles.

When we build arrays with side lengths of 6 and 3, we know there are 18 square tiles. The area is 18 square units.
Connect this tightly to multiplication using area model.
6 and 3 are the factors: meaning side length, and 18 is the area.

Build rectangles as shown above using Cuisenaire rods. The total area is the number of square units (white/grey) that it would take to cover the area of the rectangle created.
Emphasize that when we calculate area we express it in square units because we are calculating the number of squares it would take to cover the area. The square size is determined by which unit you are using. For example cm squares, metre squares, km squares etc.

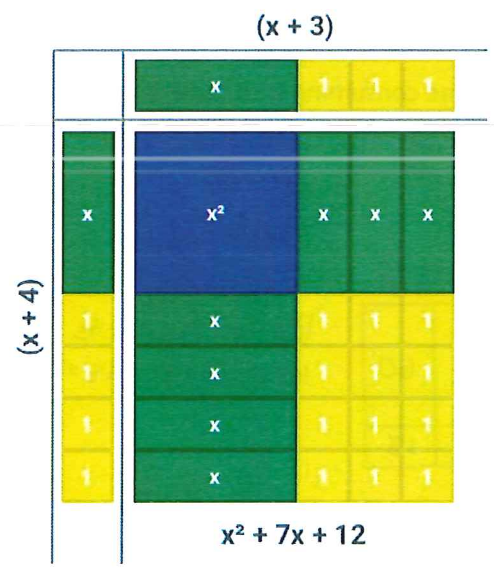
PERIMETER
Perimeter is the distance around the square/rectangle. If you were just counting the squares along the edges, how many would there be? Think of building a fence.
Make the connection that if we know the length, we can double it because there are two sides that length. If we know the width we can double it because there are two sides that length. If we add them all together we will know how far it is around the rectangle, which is the perimeter.

Perimeter of a rectangle is $2l + 2w$ or $2(l + w)$

Where does this lead?

Calculating the area of composite objects
Area and perimeter of other shapes (triangles, parallelograms, circles etc)
Surface area of 3-D objects

Grade 10: Factoring of polynomials is the area model!! Factor $x^2 + 7x + 12$ (start by making a rectangle with an x^2 tile, 7 of the x tiles and 12 unit tiles. When you've made the rectangle, the side lengths are the two factors- in this case $x + 3$ and $x + 4$



<table>
<tr><td>Curricular content

Relating multiplication facts and division facts:

<u>Division facts to 100</u> (Emerging computational fluency).
Students are expected to know how multiplication facts are used in division. E.g. 45 ÷ 9 = 5
In grade 5 students should at least know multiplication facts 2,3, 4, 5 and 10 fluently-and start making the connection to division facts.

Division to 3 digits, including division with remainders.</td>
<td>Examples and Strategies

Division is sharing equally. Two methods to teach 231 ÷ 6 =
The proper way to read this equation is 231 divided by 6, OR 231 shared 6 ways. A common misconception is to say 6 divided by 231- usually when it is written as long division format. Sometimes you also hear “goes into” which isn’t helpful to understanding that division is actually sharing.

Method 1: Division by repeated subtraction
This method allows students to work from the facts they already know. Usually this means groups of 100, 10, 5, 2 and sometimes 25 and 50. As students get better at doing it they will see that using larger groups will get them to the answer faster, but you can get the same answer even if you just take single groups the whole time.
The line down the right side of the equation shows where you record the number of groups that you are sharing. In this example, both students chose to take 10 groups of 6 in the first step. You then subtract the 60 from the 231 and you have a new total that remains to be shared. The first example shows the student decided to now take 20 groups of 6, which is 120 whereas in the second example the student stuck with their known fact that 10 groups of 6 is 60.

231 ÷ 6 =
38 r3 Groups
6) 231 | 10
-60
171 | 20
-120 | 7
51 | 1
-42
9
-6
3
add the total number of groups
= 38 r3
or 38 3/6
or 38 1/2

38r3 Groups
6) 231 | 10
-60
171 | 10
-60
111 | 10
-60 | 5
51 | 1
-30 | 1
21 | 1
-6
15
-6
9
-6
3</td></tr>
<tr><td>Language

Divisor: the number you divide the dividend by. Example- how many people you are sharing the dividend amongst.

Dividend: the total amount that you are sharing</td>
<td>Method 2: Division by decomposition
Make sure students are good at repeated subtraction division method before moving on to this.
This looks more like the traditional algorithm, and understanding is critical. Don’t just show the written procedure- make sure they understand the concept of sharing equally.
Tips for teaching (using the equation 231÷ 6=)
A) Build the number using base ten blocks and have students stand at the front of the room holding the manipulatives (one person holding two hundreds, another holding three tens and the other holding 1 one.)
B) In this case, have 6 people go to the front to try to share the manipulatives. Start with the hundreds. They won’t be able to share the hundred flat without trading it in for tens.
C) Put all the ten rods into one person’s hands (in this case, 23 ten rods). Now that person can model sharing those rods with the six people. They will each get 3.
D) The remaining 5 ten rods will have to be traded in for ones units. Put all ones units together and then share equally amongst the six people.</td></tr>
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After students really understand this, then show how that is recorded: see the example below

$231 \div 6 =$

$$
\begin{array}{r|l}
 & \underline{\;38\text{ r}3\;} = 38\tfrac{3}{6} \text{ or } 38\tfrac{1}{2} \\
6 & 231 \\
 & \underline{-180} \\
 & 51 \\
 & \underline{-48} \\
 & 3
\end{array}
$$

$214 \div 7 =$

$$
\begin{array}{r|l}
 & \underline{\;30\text{ r}4\;} = 30\tfrac{4}{7} \\
7 & 214 \\
 & \underline{210} \\
 & 4
\end{array}
$$

In this example make sure that when you show that the ones cannot be shared you record that by showing "0"

Where does this lead?

Division with decimals

$4.275 \div 6 =$

$$
\begin{array}{r|l}
 & \underline{\;0.7125\;} \\
6 & 4.2750 \\
 & \underline{42} \\
 & 07 \\
 & \underline{-06} \\
 & 15 \\
 & \underline{-12} \\
 & 30 \\
 & \underline{30} \\
 & 0
\end{array}
$$

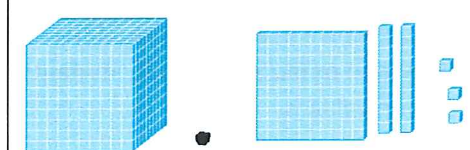
the zero is placed here in order to "finish sharing"

Grade 12 Division of polynomials

$$
\begin{array}{r|l}
 & \underline{\;x + 2xy - 3y\;} \\
x+y & x^2 + 2x^2y - 2xy + 2xy^2 - 3y^2 \\
 & \underline{-x^2 \qquad + xy} \\
 & 2x^2y - 3xy + 2xy^2 - 3y^2 \\
 & \underline{-\;2x^2y \qquad + 2xy^2} \\
 & -3xy - 3y^2 \\
 & \underline{-3xy - 3y^2} \\
 & 0
\end{array}
$$

Grade : 5

Critical concept: Fractions and decimals

Curricular content	Examples and Strategies
Decimal fractions to thousandths Benchmarks of zero, half and whole Equivalent fractions	Start with a review of place value showing each place value is ten times larger/smaller than the column next to it. There are special manipulatives you can get that are built proportionally for decimals. (Decimal strips) If you don't have these, you can use the regular base ten blocks as shown below: Name the cube "one" The flats are then tenths Rods are hundredths Unit cubes are now thousandths  1 . 123
Language Decimals are a special type of fraction- they are always expressed as tenths, hundredths, thousandths etc. They can also be called decimal fractions. (Special type of fraction where the denominator is always 100) Numerator :how many we have Denominator: how many equal parts make up a whole Fractions are made up of equal size pieces (shares of a whole) **Important for naming decimals: 0.4 is said 4 tenths, not zero point four Factor: a number that when multiplied by another factor gives you a product.	Build numbers using this model. Practice naming. Example 1.04 is one and 4 hundredths 1.205 is 1 and two hundred five thousandths Linking decimals and fractions is easy to see when you use Cuisenaire rods Model tenths using Cuisenaire rods as in the example below. Show that ten tenths is one whole.  Each of the grey/white is one tenth (because it takes ten to make a whole orange). You can show that one tenth is written 0.1 or $\frac{1}{10}$. Decimals are fractions written with a multiple of ten as a denominator. If you were to continue to put down white squares, you would have more than one whole orange rod; written as 1.1 or 1.2 etc depending on how many more white tenths you have than one whole. In addition to understanding the relative size of decimals, and knowing how to name decimal fractions, grade 5 students are required to do addition and subtraction with decimals to thousandths.

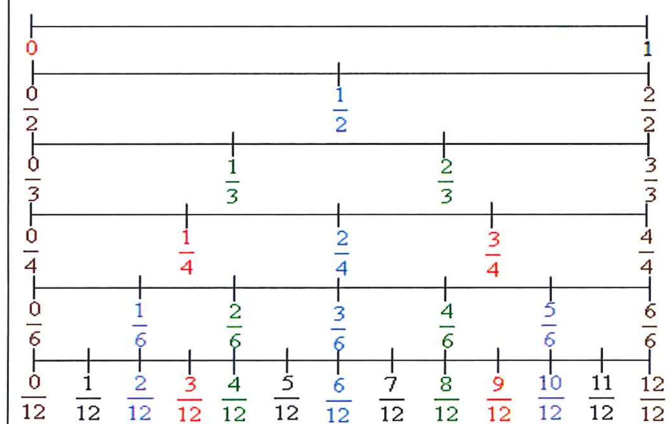
Equivalent fractions: represent the same amount, or same length, but the number of pieces is different

FRACTIONS

It is important to remember that fractions are really the first time that students will have seen a situation where the size of a "value or number" isn't fixed. For example one-half can be a different size depending on the size of the whole. (refer to the grade 4 critical concept sheet for an explanation)

Benchmarks of zero, one half and one

Using a number line:



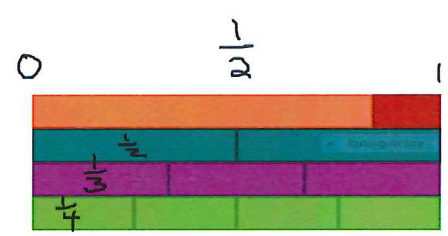
Start with a number line showing zero and one whole. It is important to emphasize that 1 means "one whole" and the values in between zero and one are when we only have a fraction, or parts, of one whole.

Dividing the number line first into half and marking the half way point will likely be easy for students to see.

Then divide the same number line into thirds, then fourths as shown in the diagram. What is essential for students to understand here is whether a given value is less than or more than half. Students can work toward being able to estimate how close to the whole, or half, a given value is.

It is not necessary to go all the way through to twelfths as shown in this diagram, but it may be useful for estimating purposes.

Benchmarks using Cuisenaire Rods



Create a "whole" – in the case of the example it is of length 12

Create trains of pieces of the same colour. In this example, dark green takes two pieces to make up the whole, meaning each piece is length ½

Purple is 1/3, light green is ¼ etc. You can now compare fractions to see whether they are closer to benchmarks of zero, half and whole.

Equivalent fractions

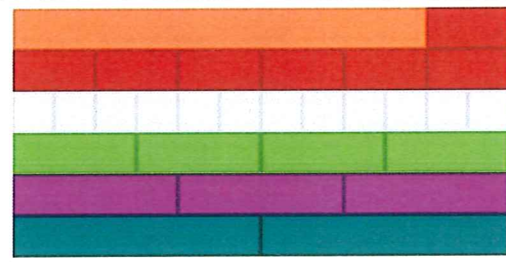
Equivalent fractions: same length, with more or less pieces

Students often find it easier to create equivalent fractions with more pieces compared to with less. E.g finding that $\frac{2}{3} = \frac{4}{6}$ is easier than finding $\frac{8}{12} = \frac{2}{3}$ Make sure you practice finding both ways.

Using Cuisenaire rods is a nice way to show that equivalent fractions are the same length. In the example below we built a whole of length 12. Then ask students to build as many FACTOR trains as they can. A factor train is all the same colour and is exactly the same length as the whole. If a number is NOT a factor, then you will not be able to make a train of the same length.

Once you have built all the factor trains, you can find places where the pieces are the same length. For example, find the trains that have pieces that end at exactly one half. Answer: $\frac{3}{6}, \frac{6}{12}, \frac{2}{4},$

You can do the same for 1/3, ¼ etc



From this diagram you can see many equivalent fractions (equivalent lengths). For example, you can see $\frac{3}{12} = \frac{1}{3}$ and $\frac{6}{12} = \frac{3}{6} = \frac{1}{2} = \frac{2}{4}$ etc

Once students are good at finding equivalent fractions using Cuisenaire rods you can start making the connection with the multiplicative relationship. (we increase or decrease the number of pieces that make up the numerator and denominator by the same factor)

Example

$$\frac{2}{3} \underset{\times 3}{\overset{\times 3}{=}} \frac{6}{9}$$

$$\frac{6}{9} \underset{\div 3}{\overset{\div 3}{=}} \frac{2}{3}$$

Where does this lead?

Rational expressions (grade 11)

$$\frac{2x}{xy} + \frac{4}{x^2} - 3 =$$

$$= \frac{2x^2+4y-3x^2y}{x^2y} \quad x \neq 0, y \neq 0$$